CSE 125
Discrete Mathematics

Nazia Sultana Chowdhury
nazia.nishat1971@gmail.com

Definition

A graph $G = (V, E)$ consists of V , a nonempty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Directed Graph

A directed graph (or digraph) (V, E) consists of a nonempty set of vertices V and a set of directed edges (or arcs) E. Each directed edge is associated with an ordered pair of vertices.

The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.

TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Some More Terminology

- Incident
- Adjacent
- Isolated
- Degree, In-degree, Out-degree
- Self-loop
- Multi edges
- Regular Graph
- Connected Graph
- Null Graph (N_n)

THE HANDSHAKING THEOREM Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} \deg(v).$$

(Note that this applies even if multiple edges and loops are present.)

An undirected graph has an even number of vertices of odd degree.

Complete Graphs

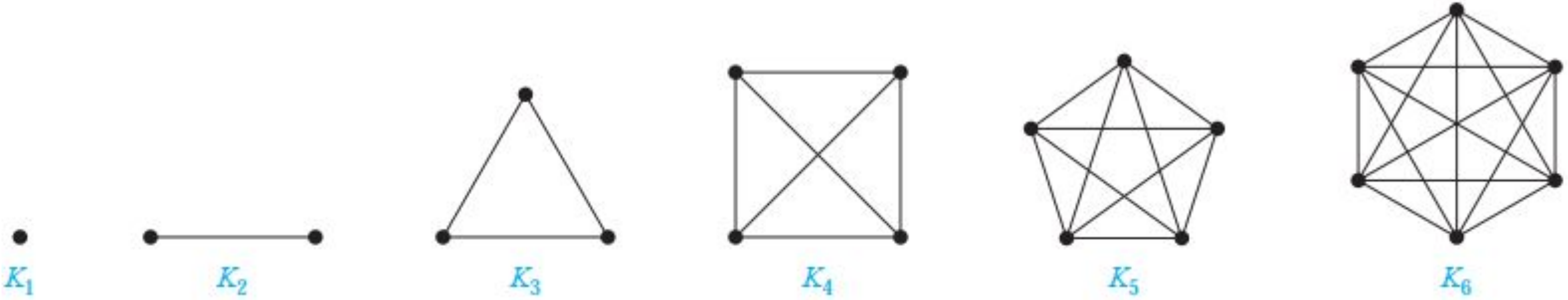


FIGURE 3 **The Graphs K_n for $1 \leq n \leq 6$.**

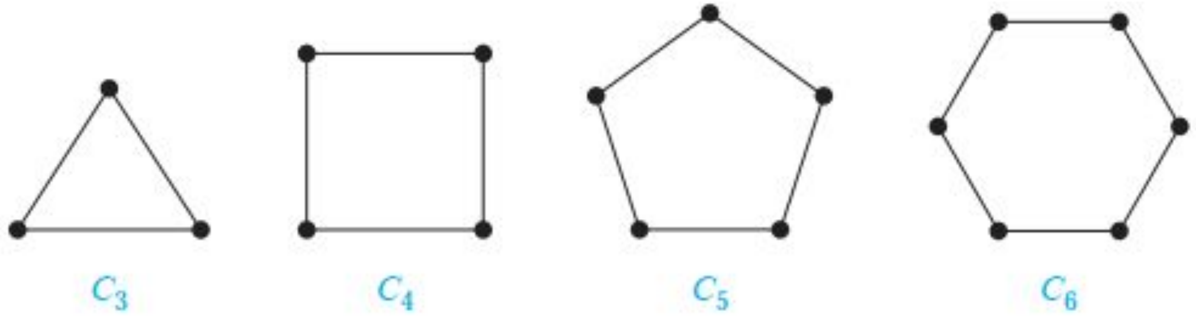
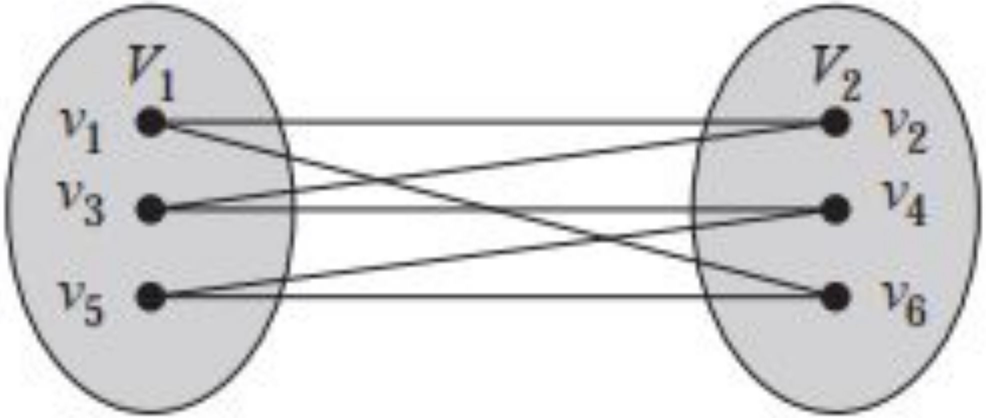


FIGURE 4 **The Cycles C_3, C_4, C_5, and C_6.**

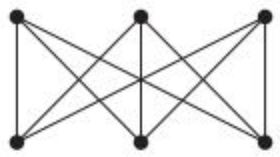
Bipartite Graph

A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

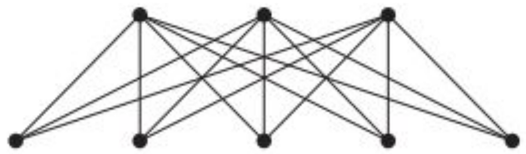


Bipartite Graph

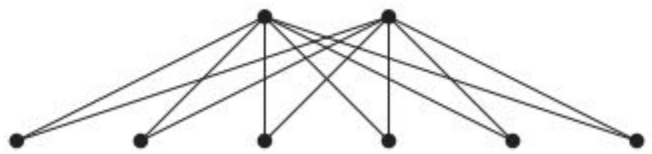
$K_{2,3}$



$K_{3,3}$



$K_{3,5}$



$K_{2,6}$

Fig: Some Complete Bipartite Graphs

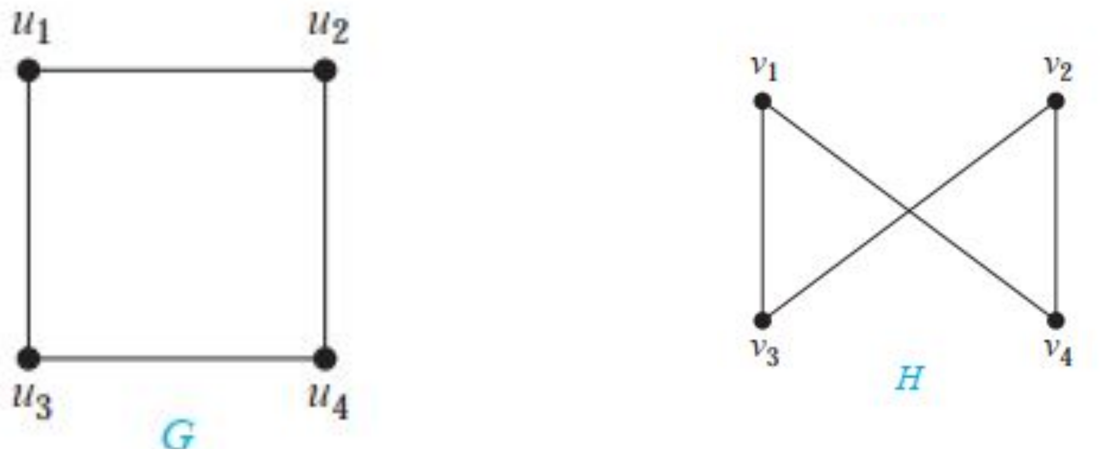


Fig: Isomorphic Graphs

Walk in Graph

- A walk is a sequence of vertices and edges of a graph.
- Vertex can be repeated.
- Edges can be repeated.

Walk in Graph

- **Open walk:** A walk is said to be an open walk if the starting and ending vertices are different.
- **Closed walk:** A walk is said to be a closed walk if the starting and ending vertices are identical.

Trail in Graph

- Trail is an open walk in which no edge is repeated.
- Vertex can be repeated.
- Closed Trails are called **Circuits**.

Path in Graph

- It is a trail in which neither vertices nor edges are repeated.
- It is also an open walk.
- Closed path is called Cycle.

Euler Path & Circuit

- An Euler circuit in a graph G is a simple circuit containing every edge of G.
- An Euler path in G is a simple path containing every edge of G.
- An Euler Graph is a connected graph that contains an Euler Circuit.
- No edges should be repeated.

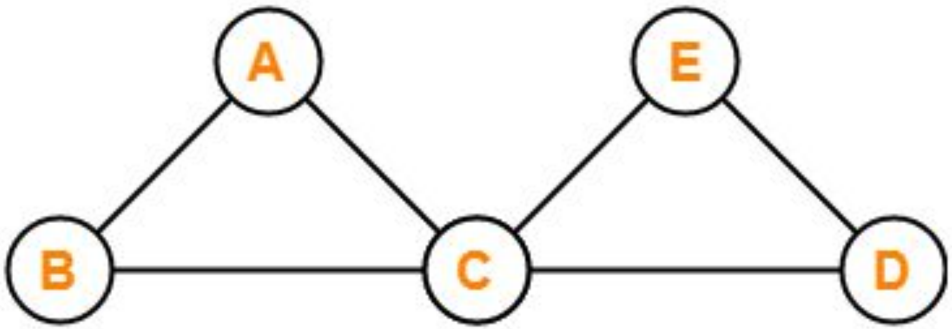


Fig: The graph contains an Euler circuit BACEDCB.

Hamiltonian Path and Circuit

A simple path in a graph G that passes through **every vertex exactly once** is called a Hamilton path, and a simple circuit in a graph G that passes through **every vertex exactly once** is called a Hamilton circuit.

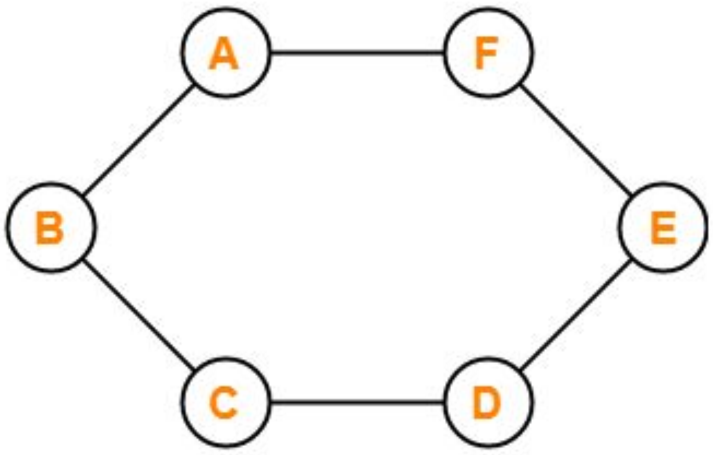


Fig: The graph contains Hamiltonian Circuit ABCDEFA

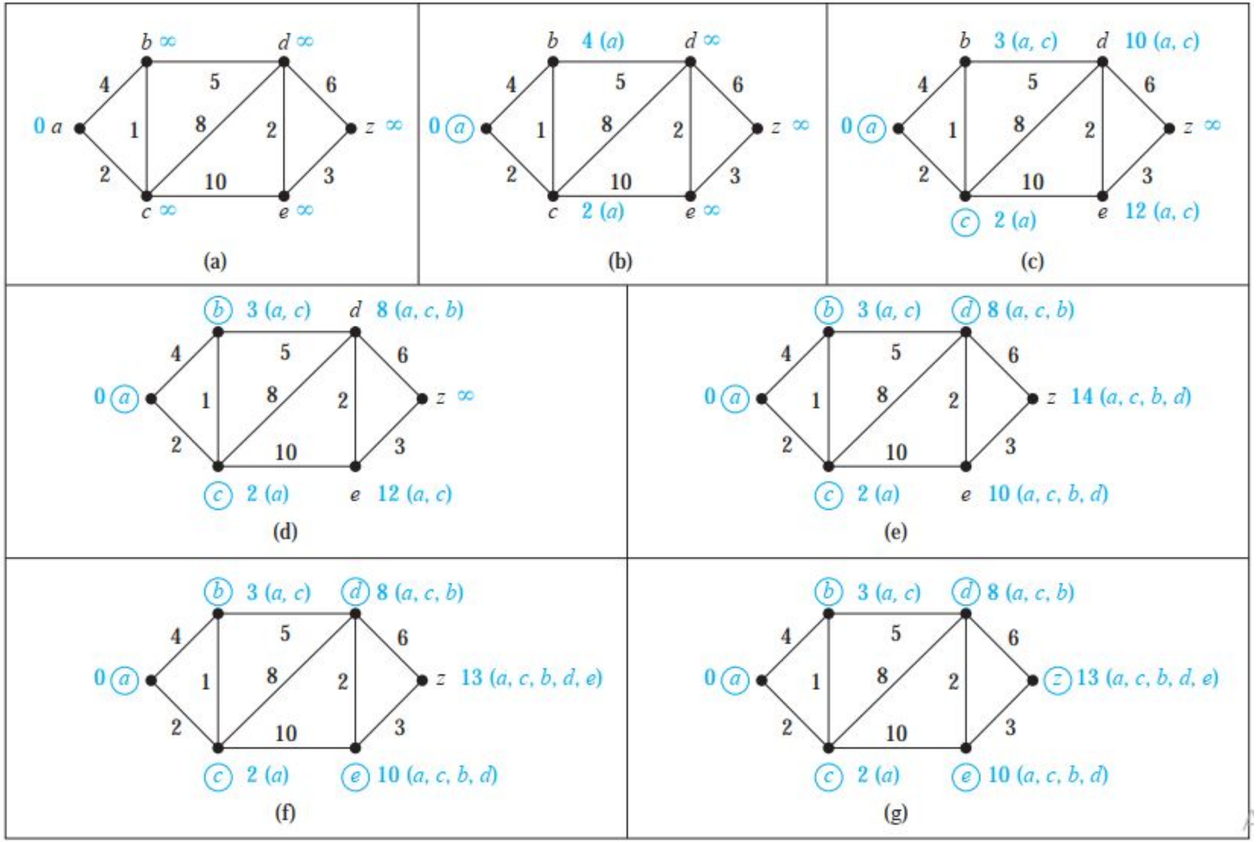
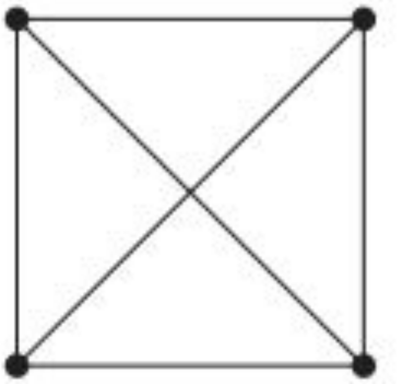


Fig: Using Dijkstra's Algorithm to Find a Shortest Path from a to z.

Planar Graph

- A graph is called planar if it can be drawn in the plane without any edges crossing. Such a drawing is called a planar representation of the graph.
- A graph may be planar even if it is usually drawn with crossings, because it may be possible to draw it in a different way without crossings.



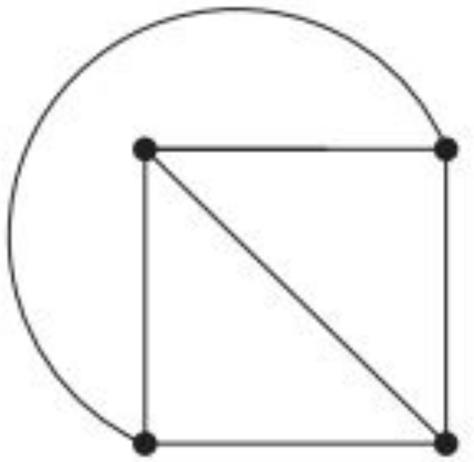


Fig: The Graph K4.

Fig: K4 Drawn with No Crossings.

Euler's Formula

- A planar representation of a graph splits the plane into regions, including an unbounded region
- Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then $r = e - v + 2$.

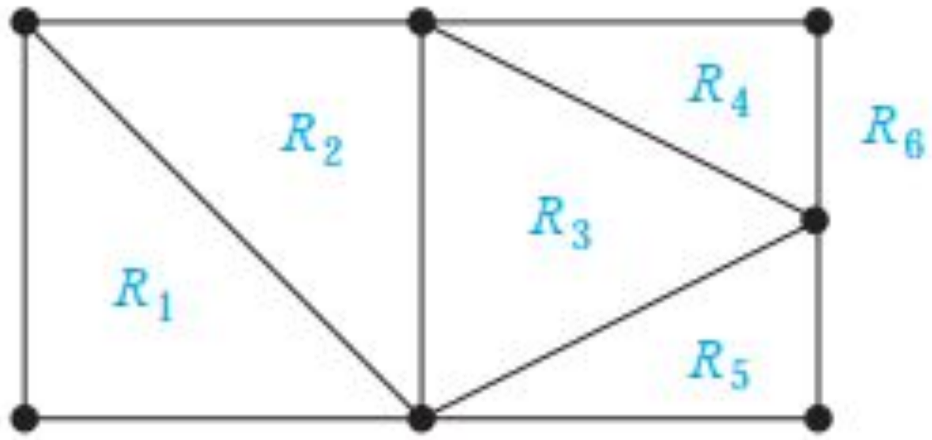


Fig: The Regions of the Planar Representation of a Graph.

Graph Coloring

- A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.
- The chromatic number of a graph is the least number of colors needed for a coloring of this graph.The chromatic number of a graph G is denoted by $\chi(G)$. (Here χ is the Greek letter chi.)

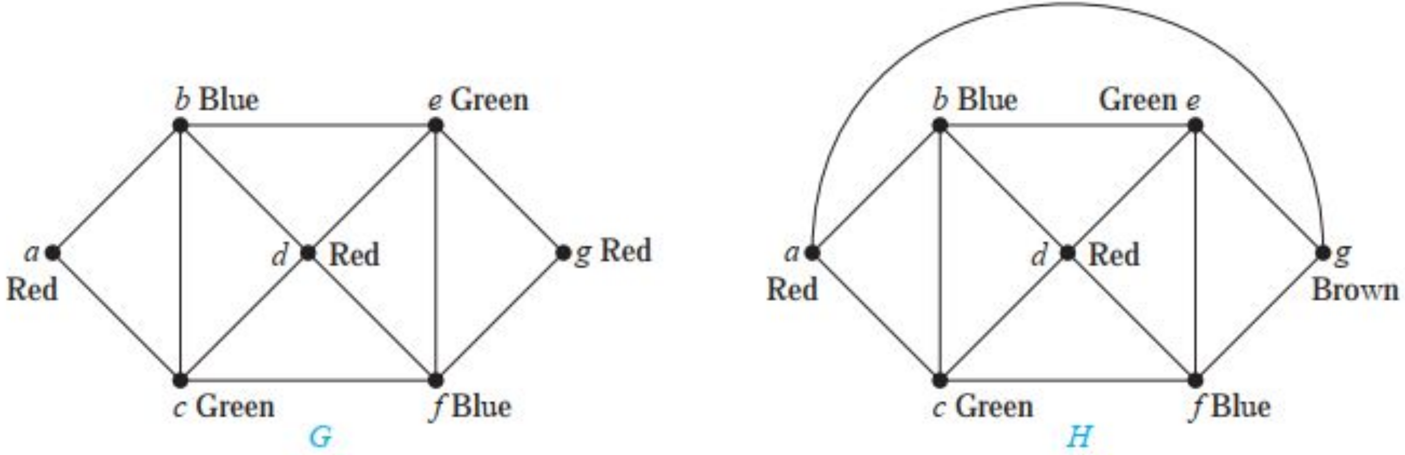


Fig: Colorings of the Graphs G and H.

Thank You